Mathematics Paper 1

<u>Structured Questions</u>

Model Paper 2025

Time Allowed: 2 hours 30 minutes
Total Marks: 100

You must answer on the question paper.

You must bring a soft pencil (preferably type B or HB), a clean eraser, and a dark blue or black pen. You will also need geometrical instruments.

Calculators are allowed.

Before attempting the paper, write your name, candidate number, centre name, and centre number clearly in the designated spaces.

Instructions for Candidates

- Answer all questions.
- Write your answer to each question in the space provided.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- You must show all necessary working clearly.
- Do not use an erasable pen or correction fluid.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Avoid writing over any barcodes printed on the paper.

Information for Candidates

- This paper consists of a total of **100 marks**.
- The number of marks assigned for every question or its parts is indicated within brackets [ ].
- A formula sheet will be provided with this paper.

Please read all questions carefully and follow the instructions exactly to ensure your responses are properly evaluated.

1

(a) Simplify:

$$\frac{3 + 2\sqrt{5}}{7 - \sqrt{5}}$$

.... [3]

(b) Solve and find the real roots:

$$2 + \frac{3}{x} = \frac{7}{\sqrt{x}}$$

.... [4]

2

The population P of a bacteria culture at time t hours is modelled by $P = 200e^{0,3t}$.

(a) Find the population after 5 hours.

$P =$**..................................** **[2]**

(b) After how many hours will the population reach 1000?

.. [3]

(c) The model is adjusted to $P = 200e^{kt}$. What does the constant k represent?

....

....

....

....

....

....

.... **[2]**

(d) Explain why this exponential model might break down for very large t. Give at least two clear reasons

....

....

....

....

....

....

....

....

....

....

.... **[3]**

3

The curve is given by $y = x^3 - 6x^2 + 8x$

(a) Find $\frac{dy}{dx}$

.... **[2]**

(b) Find the *x*-coordinates of the stationary points in exact form.

x_1=..

x_2=.. **[3]**

(c) Determine the nature of each point (maxima, minima, or inflection).

First point: ....

Second point: ....

[2]

(d) Find the equation of the tangent at $x = 1$ in the slope-intercept form.

.... **[2]**

4

The first three terms of an arithmetic progression (AP) are $2,\ 7,\ 12, \ldots$

(a) Find an expression for the nth term, a_n.

$a_n =$ ....**[2]**

(b) Find the sum of the first 30 terms.

....**[2]**

(c) The 15th term of another AP is 100 and its first term is 40. Find its common difference

.... **[2]**

5

A circle has equation $(x-2)^2+(y+1)^2=25$. A line has equation $y=mx-6$.

(a) Show that substitution leads to a quadratic in x of the form:

$$(1+m^2)x^2+(-4-10m)x+4=0$$

[3]

(b) Find the exact values of m for which the line is tangent to the circle.

$m =$ **............... , ... [3]**

(c) Give the equation of the tangent corresponding to the non-negative value of m.

.. [2]

(d) Hence, find the coordinates of the point of tangency for this tangent line.

(....................... , ...) **[2]**

(e) Without using the distance formula, work out the distance between this point of tangency and the centre of the circle.

....

....

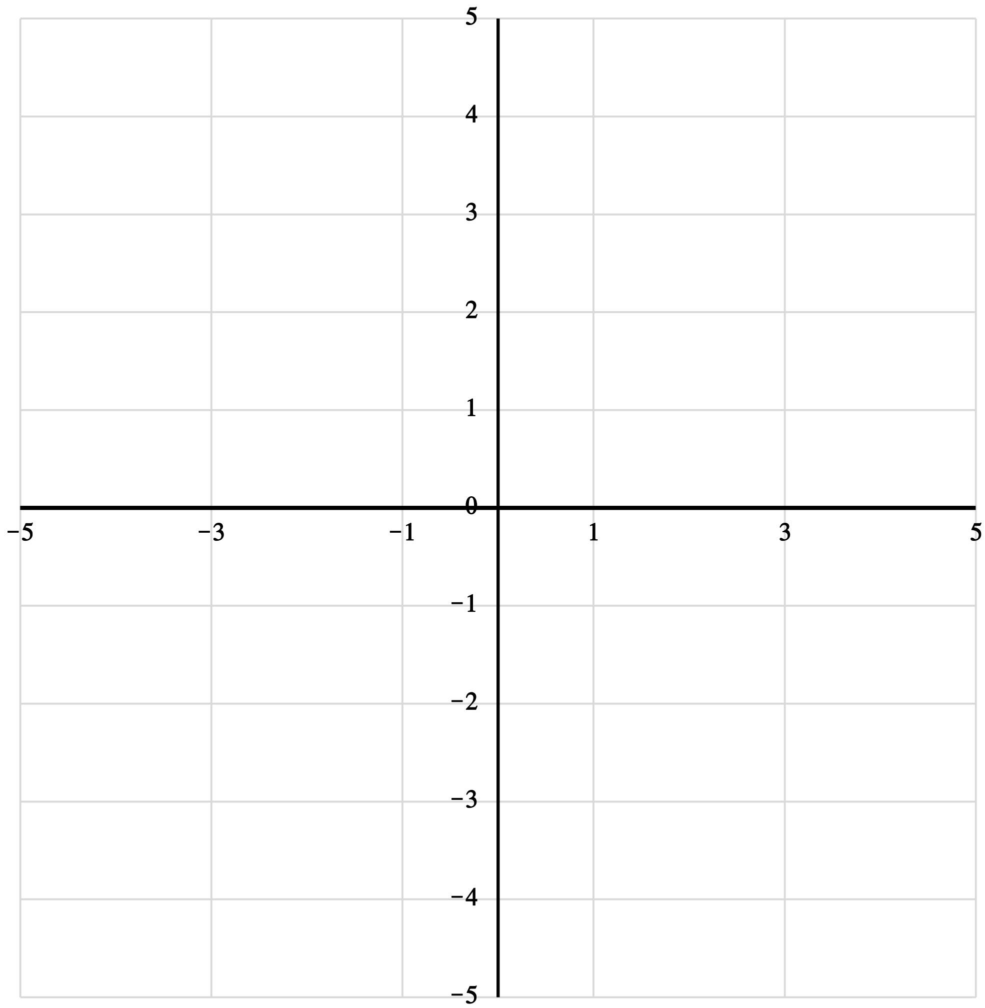
....

.... **[1]**

6

The region is bounded by the curve $y = x^2 + 2$, the x–axis, and the lines $x = 0$and $x = 2$.

(a) In the graph below, sketch the curve and shade the region. **[3]**



(b) Find exact area of the region.

Area = ... **[4]**

(c) Use the trapezium rule with 4 strips to approximate the area. Compare with the exact value.

Area= ....

Comment: ....

....

.... **[4]**

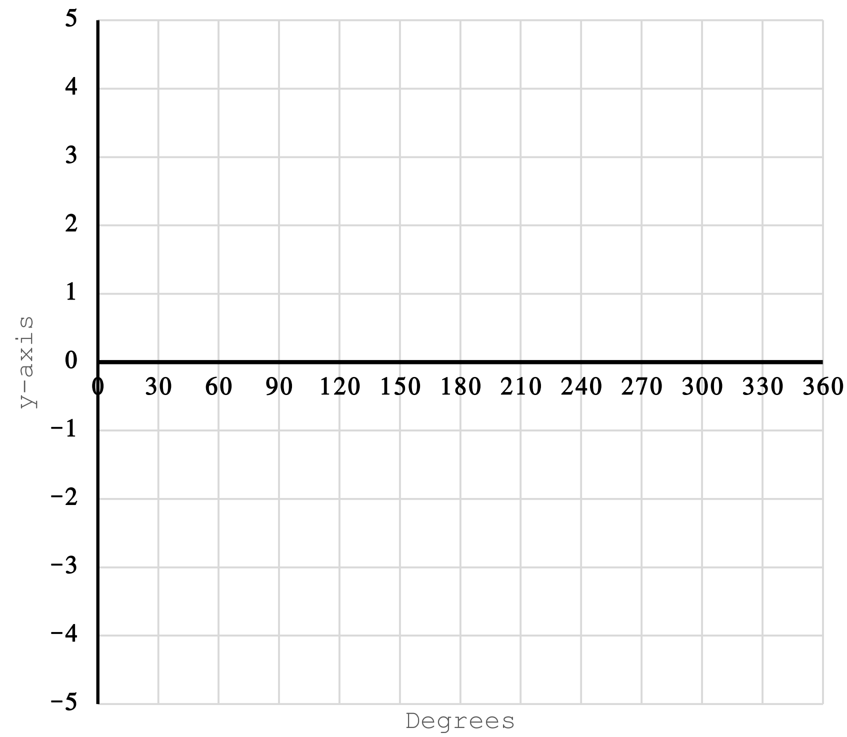
7

Lets $f(x) = \sin 2x$ and $g(x) = \cos x\,, x \in \mathbb{R}$.

(a) Solve $f(x) = g(x)$ for $0° \leq x \leq 360°$

..[5]

(b) Hence, sketch the graphs of $f(x)$ and $g(x)$ on $0° \leq x \leq 360°$ and show that when restricting the angles to $0° \leq x \leq 180°$, some solutions are excluded.



[6]

8

The expansion of $(1 - 2x)^5$ is required.

(a) Write down the general term in the expansion of T_{r+1}.

$T_{r+1} =$ **.. [3]**

(b) Find the first four terms of the expansion in the ascending powers of x.

.. [4]

9

The cubic function is defined by

$$f(x) = (x - 3)(2x - 5)(x + 1).$$

(a)

i. Expand $f(x)$

.... **[1]**

ii. Hence, verify that $f(3) = 0$.

.... **[1]**

iii. Is $(x - 3)$ a factor of $f(x)$? If yes, why?

....

....

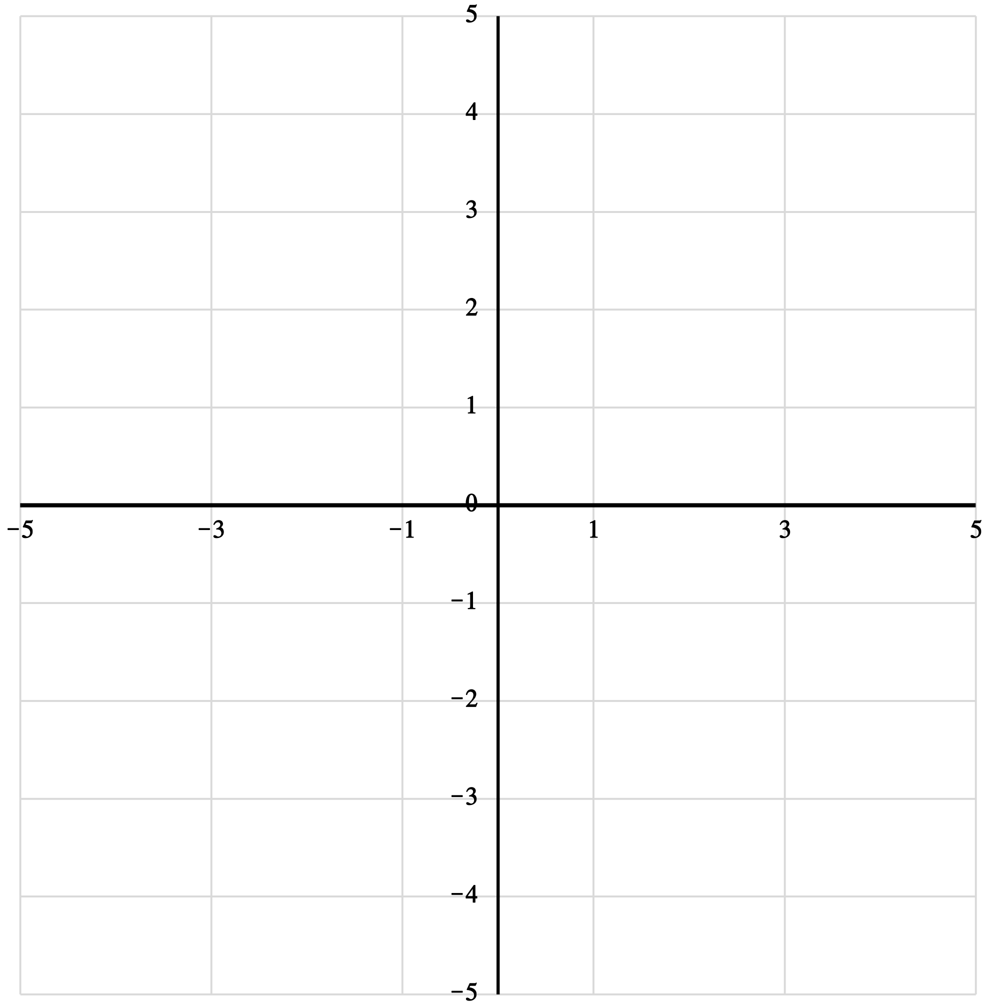
....

....

.... **[1]**

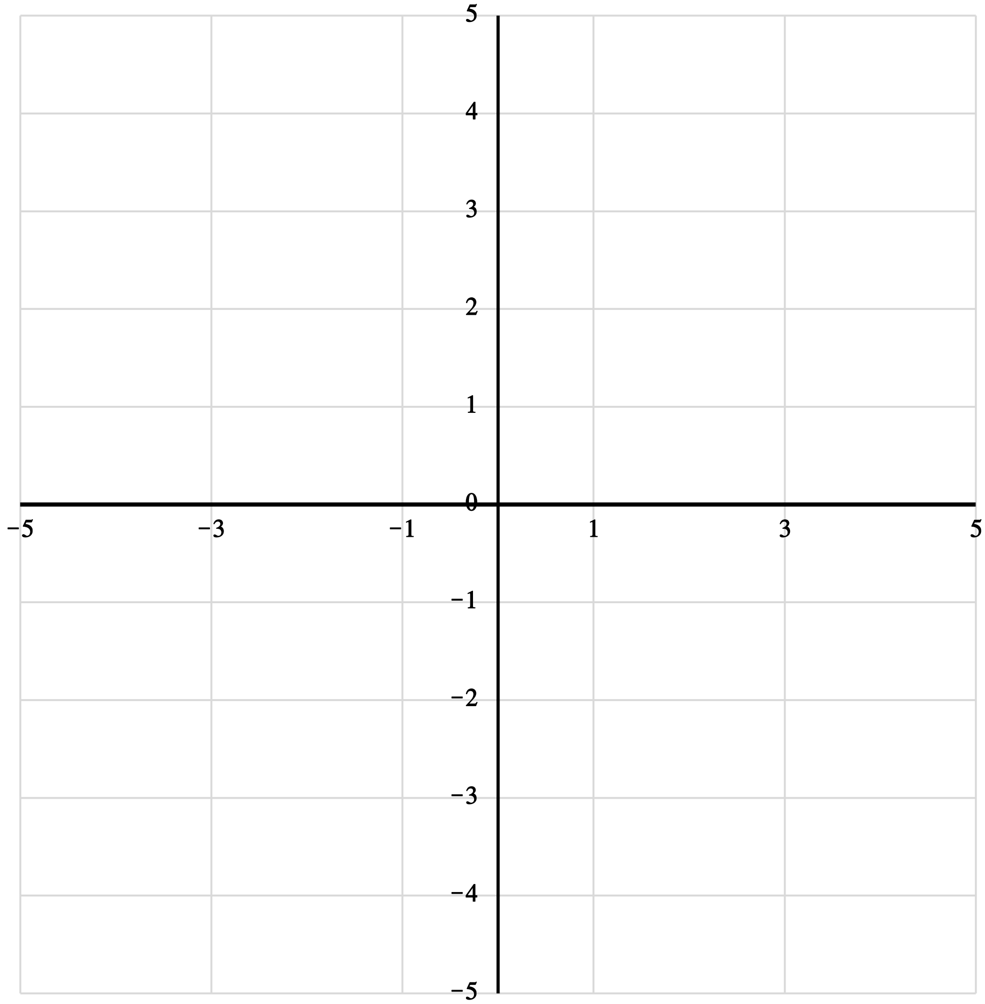
(b) Another function $g(x)$ is defined as $g(x) = (x - 3)(x + 1)$.

i. Sketch the graph of $y = g(x)$



[2]

ii. Sketch the graph of $y = |g(x)|$



[2]

iii. Describe clearly what transformations have taken place on the relevant parts of the original graph to obtain the graph of $y = |g(x)|$

....

....

....

....

.... **[1]**

10 Function: $f(x) = x^3 - 2x^2 - 5$.

(a) Show that $f(x) = 0$ has a root between $x = 2$and $x = 3$.

.... **[2]**

(b)

i. Use Newton–Raphson with initial approximation $x_1 = 2.5$ to find the root correct to 3 decimal places.

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

.... **[4]**

ii. After how many iterations should you stop? Why?

....

....

....

....

....

....

.... **[2]**

(c) Explain why Newton–Raphson might fail for some starts.

....

....

....

....

....

....

.... **[2]**

11

A function is defined by $f(x) = \sin x + \cos x$ for $x \in [0,2\pi]$.

(a) Show that $f(x) = \sqrt{2}\sin(x + \frac{\pi}{4})$. Hence solve $f(x) = 1$ for $x \in [0,2\pi]$.

.... **[3]**

(b) Are the maximum and minimum values of the function same as $\pm R$? Also give the corresponding x-values.

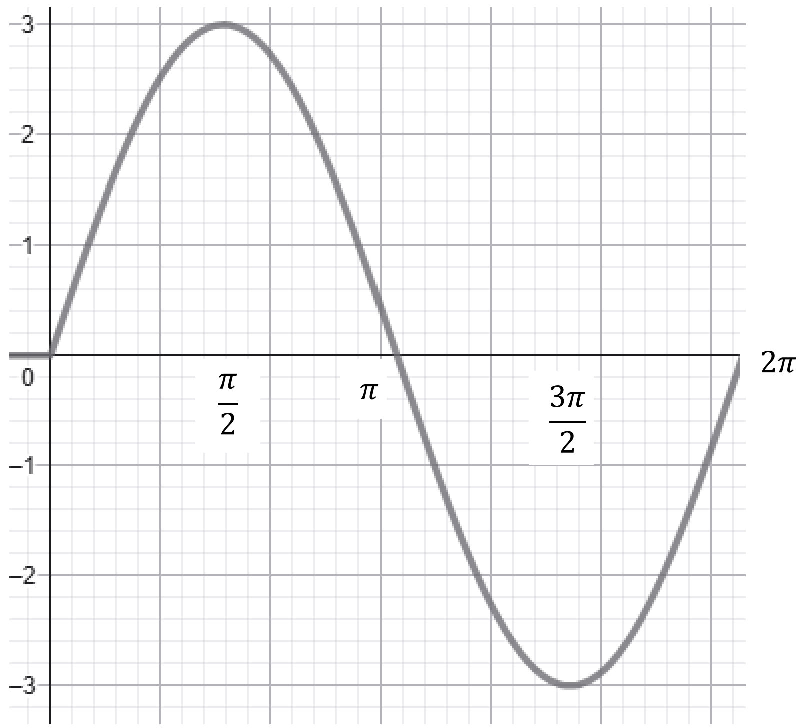
....

....

.... **[3]**

(c) The curve $y = 3\sin x$ is shown below for $x \in [0,2\pi]$.

Use integration to show that the total area between the curve and the x-axis over one complete cycle is zero. Explain clearly why the result of your integration is zero, even though the graph has visible "area" above and below the x-axis.



....

....

.... **[4]**

Model Paper 1 Marking Scheme

Question		Solution	Notes
Q1	(a) Simplify: $$\frac{3+2\sqrt{5}}{7-\sqrt{5}}$$	Multiply numerator and denominator by the conjugate $(7+\sqrt{5})$ $$\frac{(3+2\sqrt{5})(7+\sqrt{5})}{(7-\sqrt{5})(7+\sqrt{5})}$$ Denominator: $7^2-\left(\sqrt{5}\right)^2 = 49-5 = 44$ Numerator: $(3)(7)+(3)\left(\sqrt{5}\right)+\left(2\sqrt{5}\right)(7)+\left(2\sqrt{5}\right)\left(\sqrt{5}\right)$ $= 21+3\sqrt{5}+14\sqrt{5}+10$ $= 31+17\sqrt{5}$ Simplify: $$\frac{31+17\sqrt{5}}{44}$$	**M1** for multiplying by conjugate **M1** for correct expansion for numerator and denominator **A1** for simplified fraction
	(b) Solve and find the real roots: $$2+\frac{3}{x}=\frac{7}{\sqrt{x}}$$	Let $\sqrt{x}=y \Rightarrow x=y^2$ Substitute: $2+\frac{3}{y^2}=\frac{7}{y}$ Multiply by y^2 and rearrange: $2y^2-7y+3=0$ (Candidate can also use any other method to solve quadratic equation at this point) Factorize: $(2y-10)(y-3)=0$ Hence, $y=\frac{1}{2}$ and $y=3$ Recall $y=\sqrt{x} \Rightarrow y^2=x$ So, $x=\left(\frac{1}{2}\right)^2=\frac{1}{4}$ and $x=9$	**M1** for substitution **M1** for correct quadratic **M1** for correct solving quadratic equation (by any means) and back substitution **A1** for correct values of x
Q2	A The population P of a bacteria culture at time t hours is modelled by $P=200e^{0,3t}$.		
	(a) Find the population after 5 hours.	Substitute $t=5$: $P=200e^{0.3\times5}=200e^{1.5}$. $e^{1.5}\approx 4.48168907$ $P\approx 200\times 4.48168907$ $=896.337814$ $P\approx 896$ (to 3s.f)	**M1** for correct substitution **A1** for correct value of P

	(b) After how many hours will the population reach 1000?	Solve $200e^{0.3t} = 1000$. $e^{0.3t} = \frac{1000}{200} = 5$. Take natural log: $0.3t = \ln 5$. $t = \frac{\ln 5}{0.3}$. $\ln 5 \approx 1.6094379$ $\Rightarrow t \approx \frac{1.6094379}{0.3} \approx 5.364793$	**M1** for correct equation set-up **M1** for taking natural logarithm on both sides **A1** for numerical answer correct to 3 s.f.
	(c) The model is adjusted to $P = 200e^{kt}$. What does the constant k represent?	k is the instantaneous exponential growth rate per hour (the constant proportional rate of change). In this model $k = 0.3$ means the population grows at about 30% per hour (continuously compounded).	**E2** for valid interpretation
	(d)Explain why this exponential model might break down for very large t. Give at least two clear reasons	Reasons the model may break down for large t: (Any two of the following) 1. **Resource limits / carrying capacity:** Exponential growth assumes unlimited resources. In reality nutrients/space run out so growth slows — a logistic model would be more realistic. 2. **Changing environment / rate k not constant:** k may change with temperature, waste accumulation, or interventions (antibiotics), so constant-k exponential is invalid long-term. 3. **Biological constraints / death & competition:** Death rate, competition, predation or mutation alter net growth; eventually population may decline or stabilise. 4. **Mathematical issues / unbounded growth & sensitivity:** Exponential predicts unbounded population as $t \to \infty$, producing unrealistic enormous numbers; small changes in $k$produce large differences for large t. **Concise conclusion:** Therefore, the exponential model is acceptable for short-term growth but not for arbitrarily large t; a	**M1** for **o**ne valid reason why model breaks down (e.g. resource limits/carrying capacity). **M1** for second valid reason (e.g. k may change / death & competition / environmental effects). **A1** for final conclusion linking reasons to the need for a different model (e.g. logistic or time-varying rate).

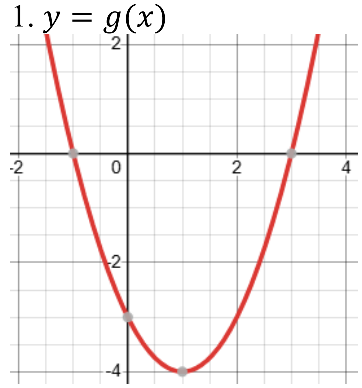
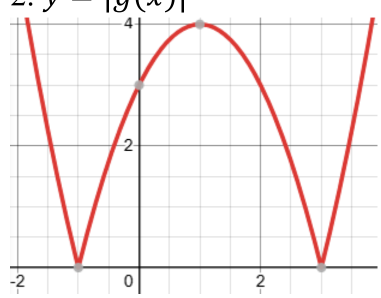
		model with carrying capacity (e.g. logistic) or time-varying k would be preferable.	
Q3	The curve is given by $$y = x^3 - 6x^2 + 8x$$		
	(a) Find $\frac{dy}{dx}$	$$\frac{dy}{dx} = 3x^2 - 12x + 8$$	**M1** for differentiating correctly each term **A1** for fully simplified derivative
	(b) Find the *x*-coordinates of the stationary points in exact form.	At stationary points, $\frac{dy}{dx} = 0$. So $3x^2 - 12x + 8 = 0$. Divide by 3: $x^2 - 4x + \frac{8}{3} = 0$. Use quadratic formula: $$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(\frac{8}{3})}}{2}$$ $$= \frac{4 \pm \sqrt{\frac{16}{3}}}{2} = \frac{4 \pm \frac{4}{\sqrt{3}}}{2}$$ $$= 2 \pm \frac{2}{\sqrt{3}}.$$	**M1** for setting the derivative equal to zero **M1** for correct use of quadratic formula/manipulation **A1** for exact *x*-values
	(c) Determine whether these points are local maxima or minima.	At $x = 2 - \frac{2}{\sqrt{3}}$: $$\frac{d^2y}{dx^2} = 6\left(2 - \frac{2}{\sqrt{3}}\right) - 12$$ $$= -\frac{12}{\sqrt{3}} = -4\sqrt{3} < 0,$$ so **maximum**. At $x = 2 + \frac{2}{\sqrt{3}}$: $$\frac{d^2y}{dx^2} = 6\left(2 + \frac{2}{\sqrt{3}}\right) - 12$$ $$= \frac{12}{\sqrt{3}} = 4\sqrt{3} > 0,$$ so **minimum**.	**M1** for evaluating correct second derivative **A1** for correctly classifying one point as maximum and the other point as minimum
	(d) Find the equation of the tangent at $x = 1$ in the slope-intercept form.	$$\frac{dy}{dx} = 3x^2 - 12x + 8$$ At $x = 1$, $$\frac{dy}{dx} = 3 - 12 + 8 = -1$$ $$y = x^3 - 6x^2 + 8x$$ $$\Rightarrow y(1) = 1 - 6 + 8 = 3.$$ Tangent: $y - 3 = -1(x - 1)$. $$y = -x + 4$$	**M1** for correct substitution in derivative to find gradient **A1** for correct equation in the slope-intercept form

Q4	The first three terms of an arithmetic progression (AP) are $2, 7, 12, \ldots$		
	(a) Find an expression for the nth term, a_n.	Common difference: $d = 7 - 2 = 5$. First term $a_1 = 2$. $$a_n = a_1 + (n-1)d = 2 + (n-1)\cdot 5 = 5n - 3.$$	**M1** for identifying the common difference and use general form **A1** for correct final term
	(b)Find the sum of the first 30 terms.	Sum of first N terms: $S_N = \frac{N}{2}(2a_1 + (N-1)d)$. Here $N = 30$, $a_1 = 2$, $d = 5$. $$S_{30} = \frac{30}{2}(2\cdot 2 + 29\cdot 5) = 15(4 + 145) = 15 \times 149 = 2235.$$	**M1** for using the sum formula correctly **A1** for the correct numerical value of sum
	(c) The 15th term of another AP is 100 and its first term is 40. Find its common difference	For the other AP: $a_{15} = a_1 + 14d$. $$100 = 40 + 14d \Rightarrow 14d = 60 \Rightarrow d = \frac{60}{14} = \frac{30}{7}.$$	**M1** for setting up the general term formula **A1** for correct value of d
Q5	A circle has equation $(x-2)^2 + (y+1)^2 = 25$. A line has equation $y = mx - 6$.		
	(a) Show that substitution leads to a quadratic in $x$of the form: $(1+m^2)x^2 + (-4-10m)x + 4 = 0$.	Substitute $y = mx - 6$ into $(x-2)^2 + (y+1)^2 = 25$: $$(x-2)^2 + (mx - 6 + 1)^2 = 25$$ $$(x-2)^2 + (mx-5)^2 = 25$$ Expand: $x^2 - 4x + 4 + m^2x^2 - 10mx + 25 = 25$. Simplify (25 cancels): $(1+m^2)x^2 + (-4-10m)x + 4 = 0$.	**M1** for substituting the equation of straight line in the quadratic equation **M1** for expanding both squares correctly **A1** for correct rearranged form
	(b) Find the exact values of m for which the line is tangent to the circle.	For tangency the quadratic has a repeated root → discriminant $D = 0$. Here $a = 1 + m^2$, $b = -4 - 10m$, $c = 4$. Compute $D = b^2 - 4ac$: $$b^2 = (-4-10m)^2 = (4+10m)^2 = 16 + 80m + 100m^2.$$ $$4ac = 4(1+m^2)(4) = 16(1+m^2)$$	**M1** for using correctly using values of *a, b* and *c* in the discriminant **M1** simplifying discriminant **A1** for correct values of m

		$= 16 + 16m^2$. $D = (16 + 80m + 100m^2) - (16 + 16m^2)$ $= 80m + 84m^2$. Set $D = 0$: $84m^2 + 80m = 0$ $\Rightarrow 4(21m^2 + 20m) = 0$ $\Rightarrow m(21m + 20) = 0$. Thus $m = 0$ or $m = -\frac{20}{21}$.	
	(c) Give the equation of the tangent corresponding to the non-negative value of m.	The non-negative value is $m = 0$. Substitute into the line: $y = (0)(x) - 6$. So, the tangent equation is: $y = -6$	**M1** for picking the right value of m and using the right equation of the line **A1** for $y = -6$ (oe)
	(d) Hence, find the coordinates of the point of tangency for this tangent line.	Find point(s) of intersection with $m = 0$: substitute $y = -6$ into the circle: $(x - 2)^2 + (-6 + 1)^2 = 25$ $(x - 2)^2 + (-5)^2 = 25$ $(x - 2)^2 + 25 = 25$ $\Rightarrow (x - 2)^2 = 0$ $\Rightarrow x = 2$. Then $y = -6$. Point of tangency: $(2, -6)$	**M1** for correct substitution and simplification **A1** for correct tangency point
	(e) Without using the distance formula, work out the distance between this point of tangency and the centre of the circle.	The point $(2, -6)$ lies on the circle whose centre is $C = (2, -1)$ and radius is 5 (from equation). Therefore, the distance CP equals the radius, which is: 5 (No distance formula required)	**B1** for correct statement that the distance equals the circle radius (5), with brief justification ("point lies on circle so its distance from centre = radius").
Q6	The region is bounded by the curve $y = x^2 + 2$, the x–axis, and the lines $x = 0$and $x = 2$.		
	(a) In the graph below, sketch the curve and shade the region.	A correct sketch will show: parabola $y = x^2 + 2$ above the x-axis, vertical lines at $x = 0$ and $x = 2$, and the region between the curve and the x-axis (shaded).	**M2** for curved sketch with correct shape and passing through (0,2) and (2,6) **B1** region correctly shaded between curve and x-axis, bounded by $x = 0$ and $x = 2$.

	(b) Find exact area of the region.	Area $= \int_0^2 (x^2+2)\ dx$. Integrate: $\int (x^2+2)\,dx$ $= \frac{x^3}{3} + 2x$. Evaluate from 0 to 2: $\left[\frac{x^3}{3} + 2x\right]_0^2$ $= \frac{8}{3} + 4 = \frac{8+12}{3} = \frac{20}{3}$.	**M1** for correct integral set up **M1** for correct antiderivative **A1** for correct substitution and evaluation at the upper limit. **A1** for correct exact value.
	(c) Use the trapezium rule with 4 strips to approximate the area. Compare with the exact value.	Step 1: width $h = \frac{2-0}{4} = 0.5$. x-values: $0,\ 0.5,\ 1.0,\ 1.5,\ 2.0$. Corresponding $y = x^2 + 2$: $y_0 = 2,\ y_1 = 2.25,\ y_2 = 3,$ $y_3 = 4.25,\ y_4 = 6$. Trapezium rule: $T = \ h\,[\frac{y_0+y_4}{2} + \sum_{i=1}^{3} y_i]$. Compute sum: $\frac{y_0+y_4}{2} = \frac{2+6}{2} = 4$. Middle sum $= 2.25 + 3 + 4.25 = 9.5$. So bracket $= 4 + 9.5 = 13.5$. $T = 0.5 \times 13.5 = 6.75 = \frac{27}{4}$. Exact area $= 20/3 \approx 6.666\bar{6}$. Difference $= 6.75 - \frac{20}{3} = \frac{27}{4} - \frac{20}{3} = \frac{1}{12} \approx 0.08333$. **Conclusion:** Trapezium approximation =6.75. It **overestimates** the exact area by $\frac{1}{12}$. The trapezium rule overestimates.	**M1** for correct determination of and evaluates y-values at $x = 0{,}0.5{,}1.0{,}1.5{,}2.0$ **M1** for applying trapezium formula correctly **A1** for comparing the exact value with the approximate value obtained by Trapezium Rule. **A1** for correct qualitative comment
Q7	Lets $f(x) = \sin 2x$ and $g(x) = \cos x\,, x \in \mathbb{R}$		
	(a) Solve $f(x) = \ g(x)$ for $0° \le x \le 360°$	Start with the double-angle identity: $\sin(2x) = 2\sin x\cos x$. $2\sin x\cos x = \cos x$. Bring to one side: $2\sin x\cos x - \cos x = 0$. Factor out $\cos x$: $\cos x(2\sin x - 1) = 0$. So either $\cos x = 0$ or $2\sin x - 1 = 0$. **Case 1:** $\cos x = 0 \Rightarrow x =$	**M1** for using the identity and rearranging to a factorable equation **M1** for correct factorization **M1** for solving the first factor correctly **M1** for solving the second factor correctly

		$90°$, $270°$(within $0°$to $360°$). **Case 2:** $\sin x = \frac{1}{2}$ $\Rightarrow x = 30°$, $150°$ (in the given interval). Final Solutions: $30°$, $90°, 150°$, $270°$	**A1** for four correct solutions
	(b) i. Hence, sketch the graphs of $f(x)$ and $g(x)$ on $0° \leq x \leq 360°$. ii. show that when restricting the angles to $0° \leq x \leq 180°$, some solutions are excluded.	Plot $f(x) = \cos x$: crosses the x-axis at $90°, 270°$; peaks at $y = 1$ at $0°$ and $360°$, trough at $y = -1$ at $180°$. Plot $g(x) = \sin(2x)$: It has period $180°$, so between $0°$and $360°$there are two full waves: zeros at $x = 0°, 90°, 180°, 270°, 360°$; positive hump from $0°$ to $90°$ etc. Mark intersections at $30°, 90°, 150°, 270°$. **Reasoning for restriction:** The solution $x = 270°$lies outside the restricted interval $0° \leq x \leq 180°$, so it is rejected. The remaining intersections within $0°$–$180°$ are $30°, 90°, 150°$. Thus, the sketch shows four intersections overall, but only three lie in $0°$–$180°$.	**M4** for sketching both curves with correct key features (zeros, peaks and intersections) **M1** for identifying the right intersections in their respective domains **A1** for correct explanation that $270°$ is outside the restricted domain so it is rejected, leaving $30°, 90°, 150°$.
Q8	The expansion of $(1 - 2x)^5$ is required.		
	(a) Write down the general term in the expansion of T_{r+1}.	The general term in the binomial expansion of $(a + b)^n$ is given by: $T_{r+1} = \binom{n}{r} a^{n-r} b^r$, where $r = 0,1,2, \dots, n$. Here $a = 1, b = -2x, n = 5$ $$T_{r+1} = \binom{5}{r}(-2x)^r$$	**M2** for substituting the correct variables into the general formula of the binomial expansion. **A1** for correct final expression
	(b) Find the first four terms of the expansion in the ascending powers of x.	for $r = 0,1,2,3$: $T_{r+1} = \binom{5}{r}(-2x)^r$. For coefficients: $$\binom{5}{0} = 1,\ \binom{5}{1} = 5$$ $$\binom{5}{2} = 10,\ \binom{5}{3} = 10.$$	**M1** for substituting correctly into the formula for correct values of x **M1** for evaluating the binomial coefficients correctly:

		Expands each term fully and correctly applies powers of $-2x$: $T_1 = 1$, $T_2 = -10x$, $T_3 = 40x^2$, $T_4 = -80x^3$.	**A1** for correct simplification and powers of x **A1** for correct final expression in simplified order
Q9	The cubic function is defined by $f(x) = (x-3)(2x-5)(x+1)$.		
	(a) i. Expand $f(x)$ ii. Hence, verify that $f(3) = 0$. iii. Is $(x-3)$ a factor of $f(x)$, if yes, why?	$f(x) = (x-3)(2x-5)(x+1)$. $f(x) = (x-3)(2x^2-3x-5)$ $= 2x^3 - 9x^2 + 4x + 15$. $f(3) = 2(27) - 9(9) + 4(3) + 15$ $= 54 - 81 + 12 + 15 = 0$. Since $f(3) = 0$, $(x-3)$ is a factor (Factor Theorem).	**M1** for correct expansion **A1** for correct substitution leading to zero **E1** for using Factor Theorem as the explanation
	(b) Another function $g(x)$ is defined as $g(x) = (x-3)(x+1)$. i. Sketch the graph of $y = g(x)$ ii. sketch the graph of $y = \|g(x)\|$ iii. Describe clearly what transformations have taken place on the relevant parts of the original graph to obtain the graph of $y = \|g(x)\|$	1. $y = g(x)$  2. $y = \|g(x)\|$  To obtain the graph of $y = \| g(x) \|$ from the graph of $y = g(x)$: The portion of the graph of $g(x)$below the x-axis (for $-1 < x < 3$) is reflected in the x-axis to produce $y = \| g(x) \|$; points above the x-axis remain unchanged.	**M2** for correctly graphing the parabola opening upwards for $g(x)$ **M2** for graphing $\|g(x)\|$ in correct shape and orientation **E1** for correct description of transformation

Q10	Function: $f(x) = x^3 - 2x^2 - 5$.		
	(a) Show that $f(x) = 0$ has a root between $x = 2$and $x = 3$.	Compute: $f(2) = 2^3 - 2 \cdot 2^2 - 5$ $= 8 - 8 - 5 = -5$. Compute: $f(3) = 3^3 - 2 \cdot 3^2 - 5$ $= 27 - 18 - 5 = 4$. Since $f(2) = -5 < 0$ and $f(3) = 4 > 0$ and $f$is continuous, there is at least one root in $(2,3)$.	**B1** for correct positive and negative values **B1** for correctly concluding that function changing sign → root exists somewhere in between
	(b) i. Use Newton–Raphson with initial approximation $x_1 = 2.5$ to find the root correct to 3 decimal places. $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$ ii. After how many iterations should you stop? Why?	$f'(x) = 3x^2 - 4x$ Iteration 1: $x_1 = 2.5$. $f(2.5) = 2.5^3 - 2(2.5)^2 - 5$ $= 15.625 - 12.5 - 5 = -1.875$. $f'(2.5) = 3(2.5)^2 - 4(2.5)$ $= 18.75 - 10 = 8.75$. $$x_2 = 2.5 - \frac{-1.875}{8.75}$$ $$= 2.5 + 0.2142857 = \frac{19}{7}$$ $$\approx 2.714285714.$$ Iteration 2**:** use $x_2 = 19/7$. $f(19/7) = \frac{90}{343} \approx 0.2627$, $$f'\left(\frac{19}{7}\right) = \frac{551}{49} \approx 11.2449$$ $$x_2 = \frac{19}{7} - \frac{90/343}{551/49}$$ $$= \frac{19}{7} - \frac{90}{3857} \approx 2.690941$$. This is accurate to 3 d.p. → **2.691** (rounded). (If desired, one further iteration will confirm no change to 3 d.p.)	**M1** for correctly evaluating $f'(x)$ **M1** for correct substitution and calculation for the first iteration **M1** for correct substitution and calculation for the second iteration **E2** for stating convergence (i.e. next iteration does not change 3 d.p.) **A1** for final root correct to 3 d.p.
	(c) Explain why Newton–Raphson might fail for some starts.	Short answers (any one acceptable): • If the derivative $f'(x_n)$ is zero or very small at an iterate, the iteration step $-f/f'$becomes very large or undefined → divergence or huge jump. • A poor starting value can lead the iterates to diverge, or be attracted	**E2** for any one correct reason/explanation

		to a different root or cycle (especially if f has inflection points, multiple roots, or the function is not well-behaved).	
Q11	A function is defined by $f(x) = \sin x + \cos x$ for $x \in [0,2\pi]$.		
	(a) Show that $f(x) = \sqrt{2}\sin(x + \frac{\pi}{4})$. Hence solve $f(x) = 1$ for $x \in [0,2\pi]$.	$\sin x + \cos x = R\sin(x + \alpha)$ Find R: square and add coefficients → $R = \sqrt{1^2 + 1^2} = \sqrt{2}$. Choose $\alpha = \pi/4$ so that $\sin x + \cos x = \sqrt{2}\sin(x + \frac{\pi}{4})$. Solve $\sqrt{2}\sin\left(x + \frac{\pi}{4}\right) = 1$ $\Rightarrow \sin(x + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$. So, $x + \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$ $\Rightarrow x = 0, \frac{\pi}{2}$ (or 2π)	**M1** for using the formula for converting sum/difference of trigonometric ratios into a single formula **M1** for calculating *R* **A1** for solving the required expression and values of *x*
	(b) Are the maximum and minimum values of the function same as $\pm R$? Also give the corresponding x-values.	Since $f(x) = \sqrt{2}\sin(x + \frac{\pi}{4})$, the maximum value is $\sqrt{2}$ (when the sine = 1) and the minimum is $-\sqrt{2}$ (when the sine = −1). Max occurs when: $x + \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$. Min occurs when: $x + \frac{\pi}{4} = \frac{3\pi}{2} \Rightarrow x = \frac{5\pi}{4}$.	**M1** for recognizing the amplitude as the max and min values **M1** for finding corresponding values of *x* for min and max **A1** for correct max and min values and their corresponding *x* values
	(c) The curve $y = 3\sin x$ is shown below for $x \in [0,2\pi]$. Use integration to show that the total area between the curve and the x-axis over one complete cycle $[0 \le x \le 2\pi]$ is zero. Explain clearly why the result of your integration is zero, even though the graph has visible "area" above and below the x-axis.	Required area: $\int_0^{2\pi} 3sinxdx = 3[-cosx]_0^{2\pi}$ $= 3(-\cos 2\pi + \cos 0)$ $= 3(-1 + 1) = 0$. The definite integral gives the net signed area: regions where $y > 0$ contribute positive area, and regions where $y < 0$ contribute negative area. Over one full cycle of $\sin x$: – From 0 to π, $\sin x$ is positive, so area = $+A$. – From π to 2π, $\sin x$ is negative, so area = $-A$.	**M1** for setting up the correct integral **M1** for finding the correct antiderivative and the overall values of integral **B1** for recognizing that integral gives signed (not geometric) are, therefore area is positive where $y > 0$ and negative where $y < 0$ **A1** for overall area of zero with correct graphical justification

		These two equal and opposite areas cancel, so the net area (integral) is 0.	